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REFINED CALCULATION OF SUPERSATURATION DURING HEAT AND MASS EXCHANGE IN A STATIONARY GAS MEDIUM

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Equations are presented for calculating the supersaturation in the absence of convection with allowance for the Stefan flow.

The appearance of a fog during the condensation of a vapor is observed in the most varied industrial and natural phenomena. In the production of sulfuric, phosphoric, and other acids, e.g., a stable and corrosive fog forms, a considerable part of which passes through the filters and is discharged into the atmosphere. The obtainment of fine powders of metals by the distillation method is accompanied by volume condensation. Processes of fog formation are also used for purposes of scientific research.

In the indicated cases the jointly occurring processes of heat and mass exchange are accompanied by a rise in the vapor pressure to values exceeding the saturation vapor pressure at the given temperature above a plane surface. The ratio $S = p/p(T)$ is called the supersaturation. But the supersaturation is limited to its critical value for each case. The value of the critical supersaturation depends both on the presence of suspended particles and on the presence of gas ions, and it can differ markedly from unity in a sufficiently purified medium [1].

When the critical supersaturation is reached the process changes qualitatively — the formation of a fog begins — and therefore in an analysis of a problem concerning volume condensation one must know whether the supersaturation has reached the critical value.

Let us consider the problem of determining the supersaturation profile in a gap formed by wet porous or solid wetted plane-parallel surfaces. To reduce to a minimum the phenomena connected with the occurrence of natural convection, we place the evaporator on top and the condenser on the bottom (Fig. 1). The gap is filled with a gas. The dimensions of the plates in the horizontal directions are so great that the problem can be treated as one-dimensional.

A simple solution to the problem under consideration was obtained in [1] on the basis of the equations $D(d^2P_1/dx^2) = 0$ and $a(d^2T/dx^2) = 0$, the integrals of which have the form

$$P_1 = P - \frac{P_e - P_c}{\delta} x, \tag{1}$$

$$T = T_e - \frac{T_e - T_c}{\delta} x. \tag{2}$$

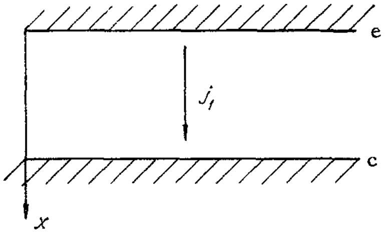

Fig. 1. Schematic representation of the process.

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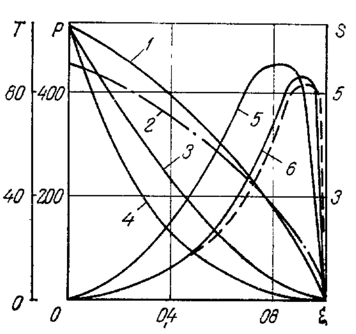

Fig. 2

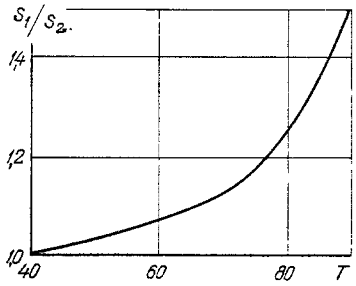

Fig. 3

Fig. 2. Dependence of parameters on the dimensionless coordinate ξ of the gap (T, °C; P_1, mm Hg): 1) partial vapor pressure found from Eq. (6); 2) temperature calculated from (3); 3) saturated vapor pressure determined from the temperature curve 2; 4) saturated vapor pressure found from the condition of linearity of the temperature distribution; 5) supersaturation calculated from the equations of [1]; 6) from (6) and (3).

Fig. 3. Dependence of supersaturation ratio on evaporator temperature at a condenser temperature of 5°C, $\xi = 0.5$, and $\delta = 12$ mm for water vapor and air; S_1) supersaturation determined by equations of [1]; S_2) supersaturation based on Eqs. (6) and (3).

Since the saturated vapor pressure is a function of the temperature, these two equations determine the supersaturation profiles in the gap. But Eqs. (1) and (2) contain only those constants taken from the boundary conditions; i.e., in the simplified variant of the solution such properties of the medium as D, C_p, etc. drop out of the analysis.

In jointly occurring processes of heat and mass exchange some part of the heat flux is expended on the heating or cooling of the moving vapor, so that the temperature profile in a medium with a stationary gas is nonlinear in the general case.

Because of the small dependence of the heat capacity of saturated water vapor and of the heat conduction of a vapor — air mixture on the temperature, the integral of the energy equation can be written like that presented in [2]:

$$T = T_e - \left(\frac{T_e - T_c}{1 - e^{g\delta}}\right)(1 - e^{gx}), \tag{3}$$

where $g = j_1 C_p/\lambda \cong \text{const}$.

In the general case the distribution of partial pressures is also nonlinear.

In our case both the evaporation surface and the condensation surface are semipermeable, so that a permeating component develops in addition to the Fick flow under the conditions of the process. This additional flow, or Stefan flow, does not play an important role when the relative humidity of the gas is low, but it can have an appreciable value at a high vapor content and affect the linearity of the characteristic curves of the process.

With allowance for the Stefan flow [3], we have

$$j_1 = \frac{P_{\text{mix}} D}{R_1 T P_2} \frac{dP_2}{dx}, \tag{4}$$

where T is a function of the coordinate.

Equation (4), after the substitution into it of T(x) in accordance with (3), is easily integrated with the following boundary conditions: $x = 0$, $P_2 = P_{2e}$; $x = \delta$, $P_2 = P_{2c}$.

In the general form we will have

$$j_1 = \frac{P_{\text{mix}} D}{R_1 \int\limits_0^\delta T dx} \ln \frac{P_{2c}}{P_{2e}}. \tag{5}$$

The distribution of partial pressures of water vapor can be obtained in general form by integrating Eq. (4), remembering that j_1 is expressed by Eq. (5) while $P_{\text{mix}} = P_1 + P_2$:

$$P_1 = P_{\text{mix}} - P_{2e} \exp \left[\frac{\int\limits_0^x T dx}{\int\limits_0^\delta T dx} \ln \frac{P_{2c}}{P_{2e}} \right]. \tag{6}$$

The integrals inside the exponential are of the same kind and differ only in the integration limits. It follows from (3) that

$$\int\limits_0^\delta T dx = \delta \left(T_e - \frac{T_e - T_c}{1 - e^{g\delta}} \right) - \frac{T_e - T_c}{g}. \tag{7}$$

Thus, a sufficiently exact solution is obtained for the problem of determining the supersaturation profile in a gap with allowance for the Stefan flow.

Equations (3), (5), and (6) presented show that the supersaturation depends on a complex of parameters.

The results of a calculation based on the functions given in the present paper in comparison with the results of [1] for a vapor – air mixture at a pressure of 760 mm Hg, an evaporator temperature of 90°C, and a condenser temperature of 5°C are presented in Fig. 2. The calculation was carried out for three sizes of the vapor – gas gap: 3, 12, and 60 mm [here the vapor flow was determined from Eqs. (5) and (7)]. For the first two gap sizes the values of the supersaturation calculated from (3), (5), and (6) as a function of the dimensionless coordinate practically coincide. The supersaturation profiles in the 60-mm gap is shown by the dashed curve in Fig. 2. The other characteristic curves of the process presented in the figure pertain to gaps of 3 or 12 mm and also practically coincide.

As seen from Fig. 2, the supersaturations found from the approximate and the refined equations can differ by 1.5 times, i.e., allowance for the Stefan correction and for the corresponding flow component on the cooling of the vapor has a marked effect on the supersaturation.

The results of a calculation of the ratio of the supersaturation computed from the results of [1] to the supersaturation found from the equations given in the present paper; as a function of the evaporator temperature at a condenser temperature of 5°C, a gap of 12 mm, and $\xi = 0.5$, are presented in Fig. 3.

As seen from the curve presented in Fig. 3, at evaporator temperatures on the order of 40°C one hardly observes a difference between the supersaturations. In the region of higher mass fluxes it is better to use equations with allowance for the additions suggested in the present paper as describing the process more precisely and carrying fuller information about it.

NOTATION

S, supersaturation; D, coefficient of molecular diffusion; P_1, vapor pressure; P_2, gas pressure; P_{mix}, pressure of mixture; x, coordinate; δ, thickness of gap; T, temperature; R_1, gas constant of vapor; j_1, vapor flux density; λ, thermal conductivity of vapor – gas mixture; C_p, heat capacity of vapor; $\xi = x/\delta$, dimensionless coordinate. Indices: e, at evaporation surface; c, at condensation surface.

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